Phase 2: Reasoning Strategies for Multiplication & Division

"Developing computational fluency is a multifaceted task that underlies all further work with numbers. During effective instruction, students need experience with a variety of ways to solve problems, time to talk about their findings, and opportunities to apply some of their ideas so they can create their own understanding of how numbers work. Computational fluency will merge as student use flexible strategies and demonstrate greater speed and accuracy" (Phillips. 2003, p. 361).

Reasoning strategies involves the students in seeking efficient strategies for the solving of basic number combinations. In so doing, students acquire the ability to look at numbers and operations in many equivalent ways. In addressing a single-digit computation, the student must first understand the problem (the meaning of the operation) and identify the quantities (numbers) with which to compute. Once this is done, strategic reasoning comes into play. Strategic reasoning involves the act of considering and selecting an efficient and effective strategy for solving the problem. At face value this seems to be a relatively simple thing to do when it comes to math facts. But, in truth, this decision-making involves two very complex questions that must be addressed by the student.

1. What computation strategies are in my repertoire?
2. How do the numbers involved relate to each other, to other important numbers in the base-10 number system, and/or numbers that are important to me?

Assumptions-
Students should have the ability to:

- Count and use groups of things as single entities
- Understand that each group consists of a given number of objects
- Represent multiplication using models, pictures and in context
- Understand and use repeated addition
- Understand and use skip counting

Fact mastery relies significantly on how well students have constructed relationships with numbers and how well they understand the operations. It is best to help students learn strategies and discover which strategies work best with what numbers. Children need lots of opportunities to make a strategy their own. It is a good idea to write new strategies on the board, chart paper, or make a poster of strategies students develop. No student should be forced to adopt someone else's strategy, but every student should be required to understand strategies that are brought to the discussion. There is a huge temptation to simply tell students about a strategy and then have them practice it. Though this can be effective for some students, many others will not personally relate to the teacher's ideas or be ready for them. Continue to discuss strategies invented in class and plan lessons that encourage strategies.

All facts are conceptually related. You can figure new or unknown facts from those you already know. "The explicit practice of strategies also requires careful selection and construction of questions that focus student's attention away from merely solving individual … facts and, instead, promote using a known fact to figure out the unknown one" (Crespo, Kyriakides, and McGee) Number relationships provide the foundation for strategies that help students remember basic facts. For example, 3 X 6 must be 6 more than 2 X 6, which is 12. So 3 X 6 is 18. 4 X 7 must be 2 X 7 twice, which is 14 twice. So 4 X 7 is 28.

Memorizing facts without understanding and/or using strategies will not help students develop relationships or be successful mastering the facts. Quick recall practice of facts should only be introduced after a deep understanding of the operation is developed and strategy use is in place. Children who are able to commit facts to memory are able to do this because they have constructed relationships among the facts and between operations. They then use these relationships as "short cuts". When relationships are the focus, students have far fewer facts to remember and have a quick way to come up with the answers.

Commutative Property of Multiplication
Understanding and using the commutative property of Multiplication is going to assist all other reasoning strategies. It allows children to apply the reasoning from other strategies regardless of the order of the factors.   This property allows you to multiply two numbers in any order and get the same product. Students do not automatically understand this property and in general that the order of the numbers does not affect the product. It would be counterproductive to introduce this property too early because it only helps in *out of context* number situations. In contextual situations the students need to understand where to attach meaning.

The array model illustrates the Commutative property.

The Zeros and Ones (Multiplication and Division)

Zero as a factor

- For multiplication, the product of any number and 0 is 0.

Zero as a divisor

- Division by 0 is not possible. "Some children are simply told 'division by zero is not allowed'. To avoid an arbitrary rule, pose problems to be modeled that involve zero.

Note: Using zero often does not occur in real life situations. It will cause difficulty for children. You need to actually model from a contextual situation to show why these rules apply.

One as a factor

- In the identity property, multiply any number by 1, or 1 by any number, the product will be that number.

> Note: Often children mistakenly apply the zero and one property of addition to multiplication and division. Using the identity property is like the zero property in that it is often difficult for children to have true understanding. They are bringing their understanding of addition and subtraction with one, which is 1 more or 1 less. In multiplication and division the number remains the same. Contextual situations can be modeled to demonstrate how zero and one as factors impact the quotient and product.

One as a quotient

- Division has an additional property. Divide a number by itself, you get one.

The set models illustrate the zero and one property.

Skip Counting (Multiplication)

Skip counting involves saying multiples of a number. This is a counting strategy. When children begin to find certain numbers are more efficient to skip count like twos, fives and tens, and utilizes this as a strategy to solve a multiplication problem, it then becomes a reasoning strategy. Using the skip counting strategy, students can model equal sized groups and count groups of items to find totals.

The number line model and/or a hundreds chart model illustrate skip counting.

Doubles (Multiplication)

Doubles is one of the earliest strategies. "Facts that have two as a factor are equivalent to the addition doubles and should be already known to students who know their addition facts." (Van de Walle. 2006. p177) For example, if a child knows 4 + 4 = 8 then this means the same, as 2 x 4 is also 8.

The set model illustrates the doubles strategy.

Derived facts (Multiplication)

Derived facts are facts learned by relating facts to an already known fact. Some of the derived facts are a combination of several strategies.

Derived facts are a limitless strategy. Listing them all is difficult and is dependent on students' individual understanding of number relationships. Using this strategy, children are actually applying the distributive property.

The array model illustrates derived facts.

> The Distributive Property of Multiplication
>
> 4 x 7 = (4 x 2 + 4 x 5)
>
> This is not found in the elementary curriculum, but in fact is the ability to compose/decompose multiple factors and to be able to multiply all of the parts. 4 x 7 could be solved by decomposing 7 and multiplying 4 x 2 and 4 x 5 and adding them together.

By decomposing a factor, a student can solve any unknown facts.
Provided are a few of the most common ways that students use known facts to help solve unknown facts.

- Double and double again applies to multiplication facts with 4 as a factor. In solving 8 x 4, the student can use 8 x 2 and then 8 x 2 again to solve.
- Double and one more set applies to multiplication facts with 3 as a factor. In solving 7 x 2, the student may know that 7 x 2 is 14 so one more set of 7 is the product of 7 x 3.
- Using tens applies to multiplication facts with 9 as a factor. In the example 3 x 9, students take the factor that is not 9 and multiply it by 10. 3 x 10 is 30. Students then subtract one from each ten, which creates a set 3 giving you the product of 27.
- Half and then double applies to multiplication of facts with even factors. The even factor is cut in half. Then the product is doubled. 8 X 6 can be halved by saying, "half of 6 eights is 3 eights which is 24, double that product of 24 to get 48."
- Add one more set applies to multiplication of a known fact that is "close" and then adding one more set to the known fact. An example would be 6 X 7, the known fact is 5 x 7, which is 35, then add one more set of 7 to get 42.

Think Multiplication (Division)
For division facts, think Multiplication! If we are trying to solve 27 ÷ 9 we would say "Nine times what is twenty-seven?" In order to solve for the missing factor, students can relate any of the multiplication strategies.

Think Multiplication (Division with a remainder)
Students will encounter many contextual situations where division will have a remainder. Students should be given ample opportunities to solve division problems that include a remainder. Students may choose any of the multiplication strategies discussed in this document to help them with a division problem that includes a remainder.

Activities for Developing Strategic Reasoning
During instruction of the strategies, a child needs time to seek the appropriate strategy for solving basic number combinations. Students should have discussion about the strategy they chose and why they selected that strategy. This helps to then transition into Phase 3 (Quick Recall). The following activities are designed to help children practice their strategies.

- Name My Numbers - Nimble with Numbers 3-4 p.116
- Melissa's Multiplication Match Up - fcpsteach
- Mama's Multipuzzle Madness - fcpsteach